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Dedicated vertical wind tunnel for the study of sedimentation of non-spherical particles

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A dedicated 4-m-high vertical wind tunnel has been designed and constructed at the University of Geneva in collaboration with the Groupe de compétence en mécanique des fluides et procédés énergétiques. With its diverging test section, the tunnel is designed to study the aero-dynamical behavior of non-spherical particles with terminal velocities between 5 and 27 m s\(^{-1}\). A particle tracking velocimetry (PTV) code is developed to calculate drag coefficient of particles in standard conditions based on the real projected area of the particles. Results of our wind tunnel and PTV code are validated by comparing drag coefficient of smooth spherical particles and cylindrical particles to existing literature. Experiments are repeatable with average relative standard deviation of 1.7%. Our preliminary experiments on the effect of particle to fluid density ratio on drag coefficient of cylindrical particles show that the drag coefficient of freely suspended particles in air is lower than those measured in water or in horizontal wind tunnels. It is found that increasing aspect ratio of cylindrical particles reduces their secondary motions and they tend to be suspended with their maximum area normal to the airflow. The use of the vertical wind tunnel in combination with the PTV code provides a reliable and precise instrument for measuring drag coefficient of freely moving particles of various shapes. Our ultimate goal is the study of sedimentation and aggregation of volcanic particles (density between 500 and 2700 kg m\(^{-3}\)) but the wind tunnel can be used in a wide range of applications. © 2013 AIP Publishing LLC. [http://dx.doi.org/10.1063/1.4805019]

I. INTRODUCTION

Transportation of solid particles within a continuum fluid is common in a wide range of phenomena. Dispersal of volcanic particles,\(^1\) sedimentation and erosion in river channels,\(^2\) deposition of solid carbon dioxide hydrate in ocean,\(^3\) particle transport in fluidized beds,\(^4\) and deposition of airborne particles in indoor environments\(^5\) are just a few examples. These phenomena are associated with various types of fluids, process speed and particle shape, size, and density. Forces and torques, which fluids exert on the particles, represent some of the most important aspects that characterize the interaction between fluids and particles.

Many natural and industrial processes involve transportation of particles either in high particle to fluid density ratios or high particle Reynolds numbers, which include many processes where solid particles are transported in gases. Drag coefficient has typically been measured of large fixed particles in horizontal wind tunnels\(^6\)–\(^9\) or of particles freely falling in liquids.\(^2\)\(^,\)\(^3\)\(^,\)\(^10\)–\(^17\) However, the measurements of the drag coefficient strongly depend on the nature of particle secondary motions, which are different for different density ratios.\(^2\)\(^,\)\(^3\)\(^,\)\(^10\)–\(^12\)\(^,\)\(^24\)

Besides, it is found that the drag coefficient of particles of any shape at intermediate Reynolds numbers (\(1 < \text{Re} < 10^2\)) is related to values of their drag coefficient at very low (\(\text{Re} \ll 1\)) and at very high Reynolds numbers (\(10^4 < \text{Re} < 10^5\)).\(^25\)\(^,\)\(^26\) Therefore, characterization of drag coefficient of particles at both high Reynolds number and high density ratios can be used either directly or to be used for estimating particle drag coefficient at intermediate Reynolds numbers. A dedicated vertical wind tunnel has been built at the University of Geneva in collaboration with the Groupe de compétence en mécanique des fluides et procédés énergétiques (CMEFE) from the University of Applied Sciences Western Switzerland in Geneva (HES-SO//hepia) (Fig. 1) in order to characterize drag coefficient of volcanic particle (i.e., highly irregular particles of various shapes, sizes, densities, and porosities). In Sec. II we first discuss fundamental aspects of forces exerted on the particle when it is freely transported in a fluid and the relationship with the particle orientation. This is followed by the methods available in the literature for the measurements of drag coefficient of particles. Advantages and disadvantages of these methods are discussed along with our motivation of building a vertical wind tunnel. Design parameters of our wind tunnel components are then presented and discussed. Our particle tracking code (PTV) developed to extract the results from the experiments with the wind tunnel is described. Finally, error estimation on our measuring method and the validation of our measurements of some spherical and cylindrical particles is presented.

II. DRAG COEFFICIENT

Particle transportation, in most cases, is associated with a fluid flow, with a fall due to gravity, with rising due to buoyancy, or with various combinations of these processes. Particles of arbitrary shapes when transported in a fluid experience forces and momentum on all three coordinate axes.\(^27\)
Magnitude of these forces is related to the shape and size of the particle, particle rotation, relative density and velocity of the particle with respect to the fluid, and fluid viscosity. As an example, when a spherical particle translates, without rotation, in a fluid with constant relative velocity the only forces acting on the sphere are buoyancy and drag. Buoyancy is a constant force which acts on the particle in opposite direction to gravity and its magnitude is equal to the weight of the fluid that is displaced by the particle volume. The drag force acts on the opposite direction to the particle motion and its magnitude is much more complex to determine. The drag force is defined as

\[ F_D = \frac{1}{2} \rho_f C_D A V_r^2, \]  

(1)

where \( \rho_f \) is the fluid density, \( A \) is a reference area which is usually chosen to be the particle projected area normal to the direction of particle motion, \( V_r \) is the relative velocity of the particle, and \( C_D \) is the drag coefficient. The drag coefficient itself is a function of particle Reynolds number, shape, and ratio of particle to fluid density. Particle Reynolds number is defined as

\[ \text{Re} = \frac{\rho_f V_r d}{\mu}, \]  

(2)

where \( \mu \) is the fluid viscosity and \( d \) is a reference length of the particle. In case of smooth spherical particles moving with constant relative velocity in an undistributed and unbounded flow, the drag coefficient is a function of Reynolds number only. Plot of drag coefficient measured in such conditions versus Reynolds number identifies the standard drag coefficient curve. \(^{28}\) For spherical particles numerous experiments are performed in a wide range of Reynolds number \(^{26,28}\) and various correlations are derived for them with a good level of accuracy, such as the model of Clift and Gauvin\(^ {8}\) which fits the experimental data with less than 6% error.\(^ {29}\) Unlike spherical particles, determination of drag coefficient of non-spherical particles is very complex and considerable amount of researches are dedicated to this category.\(^ {2,3,6,10–17,19–22,25,30–34}\)

Non-spherical particles can have either regular or irregular shapes. Regular particles are characterized by known geometrical shapes, such as circular cylinders, square cylinders, disks, and prisms. Irregular particles exist in many natural and industrial processes such as tephra transport in volcanic eruptions,\(^ {35}\) sedimentation in riverbeds,\(^ {36}\) mineral processing,\(^ {37}\) and chemical blending.\(^ {17}\)

There are several issues in the determination of the drag force of non-spherical particles. The first issue is the lack of a shape descriptor which can relate the drag coefficient of the particle to its shape. Many studies use sphericity\(^ {37}\) as a shape descriptor to correlate the shape of non-spherical particles to their drag coefficients.\(^ {25,30,33,38}\) However, beside issues of measuring and constraining sphericity of irregular particles,\(^ {35}\) McKay \etal\(^ {15}\) found that although elongated cylinders have same value of sphericity as disks, their values of drag coefficients are very different. Another issue on the measurement of drag coefficient of irregular particles is due to their secondary motion when they are freely transported in a fluid.\(^ {2,3,10,12–24}\) Secondary motions have significant effect on the drag coefficient of the particles in the direction of their primary motion.\(^ {2,3,10,12–14,23}\)

### A. Secondary motions of particles

Different types of particle secondary motion are reported for different particle shapes, ranging from small oscillation
and rotation to tumbling and chaotic motions.\textsuperscript{2,3,10,12–24} Two main sources are known to be responsible for secondary motions of particles. The first source is the way that hydrodynamical forces and torques evolve when particle degrees of freedom change. Variation in the particle state of equilibrium results in the variation of pressure and velocity fields around the particle which may help the particle to roll back to its previous state of equilibrium or may force the particle into a new equilibrium or unsteady conditions. The second source is the wake instability which occurs behind the body beyond a critical Reynolds number even if the body moves with a constant velocity and orientation. Wake instability changes the exerted vortical force and torque on the body which changes the body motion and eventually leads to a new equilibrium state in the body/fluid system. Even in case of axisymmetric bodies, when the wake behind the body loses its symmetry due to instability, an asymmetric load is produced which makes the body rotate and move sideways.\textsuperscript{24}

Secondary motions are related to particle shape, Reynolds number, and the ratio of particle to fluid density, \( S = \frac{\rho_p}{\rho_f} \). Recently, it is found that secondary motion not only exists in case of freely moving non-spherical particles but also in case of spheres.\textsuperscript{24,39,40} Secondary motions received more attention in case of non-spherical particles because they can affect the drag coefficient considerably. Reynolds number is also an important factor on the secondary motion of particles. Studies on non-spherical particles show that for most shapes when the Reynolds number exceeds value of \( \sim 100–300 \), secondary motions start.\textsuperscript{5,10,14,17,39} Increasing Reynolds number changes the type of secondary motions from small oscillations and rotation to tumbling and chaotic motion. Finally, the last parameter affecting secondary motion of the particle is the density ratio, \( S \). It is found that for non-spherical particles increasing density ratio increases secondary motions of particles which leads to a lower value of drag coefficient.\textsuperscript{3,10,12,13,29}

### B. Strategies for the measurement of particle drag coefficient

Experimental strategies represent the most accessible way to investigate the drag force of non-spherical particles freely moving in fluids. Numerical methods, on the other hand, can be performed on very basic cases, such as two-dimensional simulation of a falling cylinder.\textsuperscript{23}

Experimental studies available in the literature mostly focus on the measurement of drag coefficient of freely falling particles of regular shapes and, in particular, of circular cylinders of various aspect ratios, \( E = L/d \).\textsuperscript{2,3,10–17} In these experiments the velocity of a particle released from the top of a vertical column filled with a fluid, mostly water-based mixtures, is measured at the bottom of the column by means of optical methods when the particle reaches to a non-accelerating state. Velocity of the particle in this state is called terminal velocity, which is a constant value since the relative acceleration of particle is zero. Relative acceleration is the acceleration calculated from relative velocity of the particle and differs from particle absolute acceleration when the fluid acceleration is not zero. Zero relative acceleration is one of the conditions mentioned earlier necessary for producing standard drag coefficient curves. Previous studies\textsuperscript{28,29,41,42} indicate that measured drag coefficients of accelerating or decelerating particles are different than those measured in standard conditions. Since methods of free-fall column need a column of a certain height in order for particles to reach the state of zero relative acceleration, most researchers used liquids (e.g., water and mixtures of water-glycerin) so that a particle reaches zero relative acceleration in much lower heights compared to gas filled columns. For example, a spherical particle with diameter of 1 cm and density of 2700 kg m\(^{-3} \) needs an air-filled column of at least 130 m in order to reach 99\% of its terminal velocity (\( \text{Re} \approx 1.6 \times 10^4 \)). The same particle would reach the terminal velocity in water after 16 cm (\( \text{Re} \approx 8.2 \times 10^3 \)).

Although liquids provide the possibility to measure drag coefficient of particles in a wide range of Reynolds number, they limit the experiments at very low values of density ratios. Effect of density ratio, as mentioned previously, has a significant effect on the secondary motion of particle and on the drag coefficient. In previous studies which used falling columns filled with liquids to measure the drag coefficient of solid particles, density ratio ranged from 0.74 to 11.7.\textsuperscript{3,10,13} Some studies measured drag coefficient of particles in the air but in very low Reynolds numbers (<100) due to small height of falling columns, e.g., Wilson and Huang.\textsuperscript{21} The exception is the experiments of Christiansen and Barker\textsuperscript{12} who used a 140 m high smokestack to measure terminal velocity of spherical and cylindrical particles in the air at high Reynolds numbers (\( \text{Re} \approx 8 \times 10^6 \)). Although Christiansen and Barker\textsuperscript{12} extended the range of drag coefficient measurement to both high Reynolds numbers and high density ratios (\( S \approx 2880 \)), only drag coefficients of cylinders with aspect ratio of 1.75 and of one disk with aspect ratio of 0.225 were measured.

Falling column method is really limited to measuring drag coefficient of particles with high density ratios and high Reynolds numbers since it requires very tall columns. The other method is to use vertical wind tunnels which can suspend particles in their calibrated vertical test section. Vertical wind tunnels with open or closed circuits were mostly used to measure terminal velocity of water droplets and to study their internal circulation and freezing behavior in meteorology.\textsuperscript{43–49} In this method water droplets are released in the center of tunnel test section where the drag flow exerted by the upward air flow counterbalance droplet weight and result in free suspension of water droplets. By pre-calibration of the air flow, value of air velocity is known at the droplet stabilization point which is equal to terminal velocity of the droplet. In some wind tunnels air velocity profile in the test section is shaped to be lower at the center.\textsuperscript{45,47,48} Shaping of velocity profile is done by using different combination of honeycombs and screens, and minimizes secondary motion of droplets which helps stabilize them in the center of test section. In addition to water droplets there are some studies which used vertical wind tunnels to suspend solid particles, such as agricultural seeds\textsuperscript{50,51} and firebrands.\textsuperscript{52,53} However, a systematic way of measuring drag coefficient has not yet been achieved.

A vertical wind tunnel represents the only solution for the measurements of drag coefficient of highly irregular volcanic particles in air at high particle Reynolds numbers. Therefore,
A dedicated vertical wind tunnel was built at the University of Geneva in collaboration with the fluid mechanics group CMEFE from the University of Applied Sciences Western Switzerland in Geneva (HES-SO/hepia) (Fig. 1). The wind tunnel was designed and calibrated to measure drag coefficient of freely suspended non-spherical solid particles.

III. WIND TUNNEL DESIGN AND COMPONENTS

A maximal wind tunnel height of 4 m was allowed due to logistic reasons. Two plenums made of 15 mm thick wood are used to decrease the turbulence intensity in the tunnel circuit, one before the contraction cone and the second at the top of the diverging test section. The diverging test section is bi-dimensional and opens with a total angle of 6° (3° on each side) with two 10 mm thick plexiglass walls at back and front, and two side walls made of wood. Height of the test section is 2.7 m with minimum cross-section area of 0.31 m × 0.30 m at the bottom which increases to 0.59 m × 0.30 m at the top. A few doors at different heights on one of the test section walls have been built for inserting and removing particles. Based on the suggestion of Barlow et al., the divergence angle is small enough to avoid separation at the walls. However, to explore possibility of having separation points, tufts were taped to both side walls of the test section and no sign of separation was observed.

Velocity of air flow decreases along the test section due to the increase in the cross-section area. The velocity decrease is essential for the investigation of suspension of particles at high Reynolds numbers and particle to fluid density ratios, for which due to particle secondary motion, the suspension velocity is not constant.

The relation between the wind tunnel circuit head loss and the flow volume has been calculated using the head loss tables reported by Idel’cik. The head loss given by all circuit components, including honeycombs and grids, was added together to determine the total head loss of the wind tunnel circuit. This result leads to the selection of a high-pressure axial blower able to give 1625 Pa pressure difference for a flow volume of 15 000 m³/h, which is powered by a 15 kW asynchronous electrical motor controlled by a variable frequency drive.

A heat exchanger and a chiller are also installed at the upstream plenum ducts so as to be able to control the air temperature. The radiator fins have positive effects on the laminarization of the flow before the test section.

Plenum and screens installed at the bottom provide a homogeneous environment to distribute uniformly and equally the air flow between different sides of the contraction cone. A contraction cone (Fig. 2) is used to increase flow speed by decreasing the area and to produce a uniform velocity profile and decreasing the turbulence intensity. Ratio of the area at the cone inlet to its area at the outlet (contraction ratio) is 6.25. A wooden guiding diffuser at the top plenum channels the air flow from outlet of the test section to the fan inlet and reduces the flow turbulence. The wind tunnel at its current setup (high-velocity setup) can reach velocities between 5 and 27 ms⁻¹ and, therefore, we can suspend volcanic particles between 10 and 40 mm (5 × 10³ < Re < 8 × 10⁴, 500 kgm⁻³ < ρp < 2700 kgm⁻³). Velocities lower than 5 ms⁻¹ can be obtained when tissue filters are added at the entrance of the diverging test section in addition to using the bypass channel.

A. Airflow in the diverging test section

Previous works show that a curved velocity profile with the minimum at the center helps the stabilization of droplets. However, Knight reported that these velocity profiles do not stabilize non-spherical particles, which, due to secondary motions, frequently collide with test section walls. The effects of velocity profiles on particle suspension in our wind tunnel were investigated. A combination of honeycomb, screens, and drinking straws were used to shape the velocity profile to be lower at the center of test section. It was found that the collision of spherical particles with test section walls was significantly reduced. However, in case of non-spherical particles no improvement was achieved except for light polyfoam pyramids, which suspended stably at the center of test section with their apex downward. Therefore, we decided not to use velocity-profile shapers and to create a flat velocity profile in the test section.

Air velocity in the test section is calibrated with a Prandtl pitot tube and two micromanometers. One of the micromanometers measured the static pressure difference between the bottom of plenum and the outlet of contraction cone as a reference pressure, ΔP_ref, while the other micromanometer measured the dynamic pressure of airflow, ρ_f V_f² / 2, from a pitot tube installed in the test section. Since the experiment setup was limited to height of the light source (see Fig. 3), the test section has been calibrated up to a height of 1 m in three horizontal planes of y = 0.1, y = 0.5, and y = 1.0. A linear correlation with R² = 0.999 has been found between the reference pressure and dynamic pressure in each calibration.
planes, which is used to calculate air velocity in the cropped area (see Fig. 3):

\[
v_f = \sqrt{2 \Delta P_{ref} (0.1041 y^2 - 0.3584 y + 0.7816)/\rho_f}. \tag{3}
\]

This correlation allows us to calculate the expected air velocity within the calibrated area only by measuring the reference pressure. Boundary layer thickness, measured with Pitot tube, varies from 1 cm at the bottom of test section to 4 cm at a height of 1.5 m.

B. Experimental setup

Figure 3 shows a schematic representation of the experimental setup. In each experiment an individual particle was suspended inside the test section while being filmed with a monochrome Phantom v10 high speed camera. The camera is placed 3 m away from the wind tunnel and the lens is focused at the center of the test section. The camera lens is AF Micro-Nikkor 60 mm f/2.8D which produces low distortion images. Camera depth of field is wide enough to cover the whole regions of the divergence in the \( z \) direction. The particle is back-lit with a high frequency light source with 8 fluorescent tubes and the camera records shadow image of the particle for time periods between 2 and 3 min with the frequency of 25 fps. A paper diffuser is attached to the back wall of test section to create a homogeneous backlight.

In Fig. 3, a particle is shown in different time frames denoted by superscript \( n \). The particle moves with velocity equal to \( V^n_P \) at time \( t \), while at time \( t - dt \) its velocity is \( V^{n-1}_P \) and in time \( t + dt \) is \( V^{n+1}_P \) where \( dt (=0.04 \text{s}) \) is the time difference between successive frames. Note that particle projected area and shape change in different views and frames since the particle shape is irregular. Proper positioning of the light source leads to having different mean gray values (MGV) on the shadow image of the particle based on their position in the \( z \) direction (see Fig. 3(b)). The closer the particle to the front wall the lower the MGV of the particle (darker shadow) and vice versa. Variation of particle MGV allows us to determine the approximate position of the particle in the \( z \) direction which will be discussed later on the filters used in the PTV code.

IV. PTV CODE

The equation of particle motion in a fluid at very low Reynolds numbers (<1) is presented by Maxey and Riley \(^{55} \) and proved by Mordant and Pinton \(^{56} \) to be valid also for particles moving at high Reynolds numbers (up to \( \sim 10^4 \)). Based on the fact that the flow is incompressible in our test section \( (M < 0.1) \) and assuming the fluid velocity is steady and its components in \( x \) and \( z \) direction are zero, the equation of particle motion in the \( y \) direction reduces to

\[
m_P a_P = -(m_P - m_f)g + m_f v_f \frac{dv_f}{dy} + \frac{1}{2} \rho_f A C_D \| v_f \| v_f + F_{\text{added mass}} + F_{\text{Basset}}, \tag{4}
\]

where \( m_P \) is the particle mass, \( a_P \) is the absolute acceleration of the particle in the \( y \) direction, \( m_f \) is the fluid mass displaced by the particle \( (=\rho_f m_P/\rho_P) \), and \( v_f (= v_f - v_P) \) is the relative speed of fluid with respect to the particle in the \( y \)
direction. $F_{\text{added, mass}}$ and $F_{\text{Basset}}$ are functions of the particle relative acceleration, $a_r$. These forces are negligible for high density ratios and high Reynolds numbers, so we neglect them in our calculations.

In order to produce standard drag coefficient curves, variables in Eq. (4) need to be measured accurately. The air density and viscosity can be calculated with classic thermodynamic correlations from values logged by installed sensors in the test section measuring pressure, temperature, and relative humidity. By knowing air density, air velocity can be simply calculated at the particle position using Eq. (3).

To measure particle absolute velocity, $v_p$, absolute acceleration, $a_p$, and relative acceleration, $a_r$, a PTV code is developed. The PTV code, written in FORTRAN language, analyzes the data file extracted from an image analysis program, the data logged by sensors during the experiment, and the particle physical and geometrical properties (e.g., mass, density, and shape parameters) in order to calculate drag coefficient of the particle from Eq. (4). In order to provide the input data for the code, movies recorded during experiments are converted frame by frame to 8-bit TIFF format. Images are then cropped as shown in Fig. 3(b), so only the area which is outside of side walls’ boundary layer is used by the PTV code. A filter based on particle image MGV is developed to assure that analysis are only made on particle outside of the flow boundary layers that form near both the front and back walls. Software ImageJ is then used to analyze the cropped images and to extract a table of the particle centroid coordinates in each frame in addition to some other factors, such as the particle area, perimeter, size of bounding box, and MGV. Images where the particle was on the border of cropped area are excluded by ImageJ automatically.

The PTV code calculates particle absolute velocity, $v_p$, absolute acceleration, $a_p$, and relative acceleration, $a_r$, using three-frame polynomial fit of second order. Polynomial and spline fitting is especially important to reduce errors on velocity measurements when the tracked particle moves with acceleration. A test on the movement of a pendulum in air is setup to examine possible effects of camera recording speed on the amplification of numerical noise resulted from using fitting formulas. Maximum velocity of the pendulum was $1 \text{ ms}^{-1}$, which is in the same order of particle absolute velocity in the wind tunnel. Ratio of pendulum area to camera field of view was also in the same order to that of particles in wind tunnel experiments (1.14 pixels/mm). The pendulum test is captured and analyzed in various camera speed frequencies including 25, 50, 100, 200 fps. It is found that recording speed higher than 25 fps can result in significant amplification of numerical noise especially in the calculation of acceleration value (Fig. 4). Therefore, all movies are recorded with speed of 25 fps.

Two sets of filters are defined in the particle tracking code: acceleration filter and shadow filter. Acceleration filter is used to exclude the frames where the magnitude of relative and absolute acceleration of the particle is greater than $0.05 \times g$. This filter assures that the results have the required conditions of standard drag coefficient curves. The shadow filter is used to exclude the frames where the particle is near the front and back walls of the test section. In near wall regions the air velocity is not known due to formation of flow boundary layers, and, therefore, the drag coefficient cannot be calculated.

To calibrate the shadow filter, MGV of some selected particles of different shapes and sizes in different places of the cropped area and different perspective positions are measured. The cropped area is divided into three vertical and three horizontal intervals which leads to having 9 grid zones. Then each selected particle is kept fixed in the middle of grid zones in the $xy$ plane and in three positions in perspective direction, i.e., attached to the front wall, center, and attached to the back wall. It is found that the minimum gray value, $MGV_{\text{min}}$, occurred when the particle was attached to the front wall, and the maximum gray value, $MGV_{\text{max}}$, is measured when the particle is attached to the back wall. The calibration experiments suggested that if the MGV of the particle in zone $i$ is greater than $1.4 \times MGV_{\text{min}}$ and is less than $0.85 \times MGV_{\text{max}}$,
the particle is in the center of the test section in perspective direction. In the experiments, however, it is not possible to pre-calibrate MGV\textsubscript{max} and MGV\textsubscript{min} of each single particle in all grid zones. Therefore, MGV\textsubscript{max} and MGV\textsubscript{min} are approximated after the experiment by analyzing distribution of particle MGV during its suspension in the wind tunnel and assuming that MGV\textsubscript{max} and MGV\textsubscript{min} of the particle in each grid zone occurred when the particle was near back and front walls of the test section in that grid zone.

In Eqs. (1) and (4), \(A\) can be any reference area of the particle. However, drag coefficient of bluff bodies in high Reynolds number is mainly pressure drag which is affected by the cross-sectional or projected area, \(A_{\text{projected}}\) of the particle. In addition to drag coefficient, knowing the projected area helps us to understand better the secondary motion of particles and the existence of preferred orientations and their effects on the particle drag coefficient. Installing a second camera and another backlight source in the test section could be a solution. However, some fallbacks are identified: (1) extra cost associated with acquisition of a second camera, (2) increase of data preparation time associated with synchronization of all devices, and (3) generation of high level turbulence and unsteadiness of airflow related to the fact that either the camera or the light source should be positioned upstream.

A more convenient solution for measuring particle projected area, \(A_{\text{projected}}\), is to use computer vision algorithms. In this method, a database of particle image in two perpendicular views, e.g., front and top, is needed for each particle. A 3D-model of particles scanned by a 3D-Scanner is used to create a database of particle image in 500 random orientations and in two front and top views (result in total 1000 images). The 3D-models were originally created for measuring shapes of the particle and correlating them to their measured drag coefficients. The random-orientation database then is analyzed by ImageJ\textsuperscript{57} to extract various factors of particle image, such as area, perimeter, Feret’s diameter, and angle of major axis in each orientation. Finally, the PTV code uses the results of database analysis to find the closest match between particle image in the wind tunnel experiments and the database by comparing different factors such as area, angle of major axis, circularity, and aspect ratio (e.g., Fig. 5).

Besides calculating the particle drag coefficient experimentally, the PTV code uses random-orientation database and particle 3D-model to calculate various shape descriptors introduced by previous researchers (e.g., circularity, Corey shape factor, sphericity) and calculates the particle drag coefficient in a condition similar to that of the wind tunnel (e.g., similar air density and viscosity) with both known spherical and non-spherical models (e.g., models of Clift and Guavin,\textsuperscript{28} Wilson and Huang\textsuperscript{21} and Ginsel).\textsuperscript{25}) This leads to having a database of the particle shape descriptors and estimation of various models for each single particle that can be used for future investigations.

V. ESTIMATION OF ERRORS

In order to estimate the errors in the calculation of drag coefficients, the standard deviation of variables used in Eq. (4) should be known. Air velocity is calculated by Eq. (3) with maximum standard deviation of 2\% including fitting and repeatability errors. Validity of air velocity measurements is verified by using various flow measurement units, such as micromanometer, precision pitot tubes, and fan anemometers.

Centroid position of a particle is measured with ImageJ\textsuperscript{57} by pre-setting particle shadow threshold in order to segment particle boundaries from background. Due to the light scattering, particle shadow has soft edges and it may result into measuring lower projection area in some frames. The area reduction is almost uniform with a mean of 15\% and a standard deviation of 5\%. However, the area reduction is uniform around the perimeter of a particle shadow and should not affect the measured position of the particle centroid. The errors associated with the polynomial fit in the calculation of particle acceleration and velocity cannot be estimated explicitly but they are likely negligible since their accuracies are in the order of \(dt\) or \(dt^2\). Due to the lens perspective effects, an uncertainty on the particle position in the \(y\) direction exists. To measure this uncertainty some guidelines are inserted in the
test section at various heights and their height deviations at their ends are measured. The measurements showed that this uncertainty can result into a maximum error of 0.3% when calculating \( v_f \) at the particle position. The errors on the particle absolute velocity and acceleration resulted from lens perspective effects are investigated statistically by assuming that the distribution of particle velocity in the \( z \) direction is similar to the distribution of \( u_p \). The average error is 3% for velocity and 7% for acceleration calculations.

Effects of wall on the particle suspension velocity, on the other hand, should be investigated more in detail. Previous studies showed that measured suspension velocity of particles in bounded mediums is lower than those measured in unbounded mediums.\(^{61-67}\) This reduction in high Reynolds numbers is a function of particle cross section area to cross section area of the suspension column, \( \lambda = A_p/A_c \). Awbi and Tan\(^{64}\) measured drag coefficients of spheres in the range of \( \lambda \) greater than 0.06 in a square cross section wind tunnel and Reynolds number between \( 10^4 \) and \( 2 \times 10^5 \). Comparisons of standard drag coefficient of spheres and those measured by Awbi and Tan\(^{64}\) show that the wall effect is negligible for \( \lambda \) of 0.06 and it becomes more considerable as \( \lambda \) increases. In the case of our experiments, however, \( \lambda \) is less than 0.03 (80 mm\(^2\) \(< A_p < 3000 \) mm\(^2\), 0.09 m\(^2\) \(< A_c < 0.11 \) m\(^2\)) and wall effects on the suspension velocity and drag coefficient are completely negligible.

Our method of calculating particle projected area is benchmarked by creating images of a particle from a random-orientation database with same scaling ratio as the real experiments and estimating the particle projected area using another random-orientation database of the same particle. A Gaussian distributed random noise with a mean of 15% and a standard deviation of 5% is added to the scaled database in order to synthetically create inhomogeneity in the projected image of the particles. This allows us to investigate on the effects of lens distortion and area reduction resulted from particle shadow thresholding. The benchmarks showed that our computer vision algorithm can estimate projected area of regular particles with the average error of 3% (median error of 0.7%) and 8% for irregular particles (median error of 7%). This shows that our method is reliable and precise in case of regular particles while in case of irregular particles it has an acceptable accuracy, i.e., the results are accurate enough for studying secondary motions of particles and their preferred orientations but uncertainties may exist on the value of projected area. However, it is common to calculate drag coefficients of irregular particles based on the projected area of volume-equivalent sphere of the particle as the reference area. As a result, in case of irregular particles, we will statistically investigate the preferred orientations of the particle to assess the most likely projected area.

### VI. VALIDATION

To check the validity of wind tunnel calibration and PTV code, drag coefficients of some spherical and cylindrical particles are measured. Physical properties of the particles and wind tunnel conditions are listed in Table I. Spherical and cylindrical particles are chosen for the validation of our experimental setup since several studies exist on drag coefficient of particles of these shapes.

Figure 6 shows measured values of relative velocity of particle S5 and particle C2 resulted from the PTV code. Standard deviation of measured relative velocity before applying filters for Particle S5 is 0.34 m s\(^{-1}\), which reduces to 0.26 m s\(^{-1}\) after applying filters. For particle C2 these values are 0.57 m s\(^{-1}\) and 0.53 m s\(^{-1}\), respectively. By applying the filters, data range is reduced from 2.0 m s\(^{-1}\) to 1.1 m s\(^{-1}\) and from 3.5 m s\(^{-1}\) to 2.4 m s\(^{-1}\) for particle S5 and particle C2, respectively. Experiments on each particle are repeated at least three times to check repeatability of measurements. Two of the experiments are performed in a same reference pressure and the third one in a different reference pressure. Average of relative standard deviation of the measured drag coefficient between repeated experiments of all particles is 1.7% with the maximum of 2.0%.

A comparison of measured drag coefficient of spherical particles in the wind tunnel with those reported in the literature\(^{8,12,28,68}\) is presented in Fig. 7. Average deviation of mean of measured velocity and drag coefficient in our wind tunnel with respect to model of Clift and Guvvin\(^{28}\) is 1.8% and 3.6%, respectively, which shows the accuracy of wind tunnel calibration and reliability of the measurements.

Drag coefficients of cylindrical particles measured in the wind tunnel are also compared with previous studies (Fig. 8), i.e., data of Wieselsberger\(^{6}\) based on experiments on fixed cylinders with two free ends in a wind tunnel, data of freely falling cylinders in water by Isaacs and Thodos\(^{13}\), and cylinders freely falling in air by Christiansen and Barker\(^{12}\). It can be seen from Fig. 8 that the drag coefficient of long cylinders (two-dimensional) is considerably higher than for short cylinders (three-dimensional), which shows that the flow around cylinders with finite aspect ratio is completely three-dimensional and cannot be approximated by measurements on two-dimensional (long) cylinders. In addition to that, drag coefficients of cylinders with finite aspect ratio fixed in a wind tunnel and freely falling cylinders in water are higher than those measured by Christiansen and Barker\(^{12}\) and our wind tunnel. This shows, first, that the secondary motions of particles during their suspension in a fluid decrease their drag coefficient compared with when they are fixed in a wind tunnel. Second, the effect of density ratio is very important on the measured drag coefficient. Drag coefficients measured in air (high \( S \)) are lower (\( \sim 30\% \)) than those measured in water.

<table>
<thead>
<tr>
<th>ID</th>
<th>Shape</th>
<th>( d ) (cm)</th>
<th>( L ) (cm)</th>
<th>( \rho_f ) (kg m(^{-3}))</th>
<th>( \rho_p ) (kg m(^{-3}))</th>
<th>( \mu \times 10^5 ) (Pa s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>S1</td>
<td>Sphere</td>
<td>3.1</td>
<td>...</td>
<td>766</td>
<td>1.12</td>
<td>1.88</td>
</tr>
<tr>
<td>S2</td>
<td>Sphere</td>
<td>3.9</td>
<td>...</td>
<td>403</td>
<td>1.14</td>
<td>1.86</td>
</tr>
<tr>
<td>S3</td>
<td>Sphere</td>
<td>3.9</td>
<td>...</td>
<td>82</td>
<td>1.15</td>
<td>1.84</td>
</tr>
<tr>
<td>S4</td>
<td>Sphere</td>
<td>6.0</td>
<td>...</td>
<td>25</td>
<td>1.15</td>
<td>1.85</td>
</tr>
<tr>
<td>S5</td>
<td>Sphere</td>
<td>6.2</td>
<td>...</td>
<td>148</td>
<td>1.15</td>
<td>1.85</td>
</tr>
<tr>
<td>C1</td>
<td>Cylinder</td>
<td>2.0</td>
<td>2.1</td>
<td>636</td>
<td>1.13</td>
<td>1.87</td>
</tr>
<tr>
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<td>2.0</td>
<td>4.7</td>
<td>650</td>
<td>1.13</td>
<td>1.87</td>
</tr>
<tr>
<td>C3</td>
<td>Cylinder</td>
<td>2.0</td>
<td>8.0</td>
<td>643</td>
<td>1.12</td>
<td>1.88</td>
</tr>
</tbody>
</table>
FIG. 6. Relative velocity, \( v_r \), of particles calculated by the PTV code before and after applying shadow and acceleration filters. (a) Relative velocity of particle S5 (standard deviation and range are 0.34 ms\(^{-1}\) and 2.0 ms\(^{-1}\) before applying filters, and 0.26 ms\(^{-1}\) and 1.1 ms\(^{-1}\), after applying filters); (b) relative velocity of particle C2 (standard deviation and range are 0.57 ms\(^{-1}\) and 3.5 ms\(^{-1}\) before applying filters, and 0.53 ms\(^{-1}\) and 2.4 ms\(^{-1}\), after applying filters).

(low \( S \)). This is similar to findings of previous studies which indicated that higher density ratios result into having lower drag coefficients for freely suspended particles.\(^3\),\(^10\),\(^12\),\(^13\),\(^29\)

Chow and Adams\(^3\) investigated the effect of cylinder density ratio and aspect ratio on its drag coefficient and provided an approximate analytic solution for estimating drag coefficient of cylindrical particles in high \( \sqrt{S/E} \) values. They approximated that the distribution of pressure force exerted on cylinders in free fall is similar to that measured by Fage and Johansen\(^69\) on a fixed inclined flat plate. Then by approximating oscillation angle of a cylinder during its fall, Chow and Adams\(^3\) estimated actual particle projected area of freely falling cylinders. Based on their approximate solution, the drag coefficient of cylindrical particles with \( \sqrt{S/E} > 1.5 \)

FIG. 7. Comparison of drag coefficient of spherical particles measured in the present study with those reported in literature. The measurements of the present work are shown by boxplots: the ends of the bars represent the smallest and the largest measurements, the box thickness indicates the first and the third quartiles and the horizontal line is the median (second quartile) of the measurement.

FIG. 8. Comparison of drag coefficient of cylindrical particles measured in the present study with those reported in literature using \( Ld \) as \( A \) in the calculation of drag coefficient; Wieselsberger\(^6\) results are from wind tunnel measurements on fixed cylinders with two free ends; Christiansen and Barker\(^12\) data are from measurements on cylinders \((E = 1.75)\) falling freely in the air \((1000 < S < 2800)\); Isaacs and Thodos\(^13\) data are from measurements on cylinders \((E = 2.0)\) falling freely in the water \((1.05 < S < 11.27)\).
FIG. 9. Variation of drag coefficient of cylindrical particles with respect to \( (S/E)^{0.5} \) measured in the present study with those reported in literature using \( Ld \) as \( A \) in the calculation of drag coefficient; values of \( (S/E)^{0.5} \) for particles C1, C2, and C3 are 23, 16, and 12, respectively; Chow and Adams’ experimental data are from measurements on cylinders \((2 < E < 100)\) falling freely in the water \((1.1 < S < 8.5)\).

should converge to a value of \( 2/\pi \). Figure 9 shows that our measurements of cylindrical particles are in close agreement with the approximate solution of Chow and Adams.\(^3\) In addition, we can see how for low values of \( \sqrt{S/E} \) drag coefficient of cylindrical particles decreases as \( \sqrt{S/E} \) increases, while, for high values of \( \sqrt{S/E} \), drag coefficient is almost independent of this parameter.

To investigate the effect of the cylinder aspect ratio, \( E \), on secondary motions and projected area, histograms of area ratio, \( A^{*} \), for each particle after applying the filters is presented in Fig. 10. \( A^{*} \) is defined as

\[
A^{*} = \frac{A_{\text{projected}} - A_{\text{min}}}{A_{\text{max}} - A_{\text{min}}},
\]

where \( A_{\text{min}} \) and \( A_{\text{max}} \) are minimum and maximum projected area of the particles, respectively, and \( A_{\text{projected}} \) is the actual projected area of the cylinder calculated by the PTV code with the method described earlier. In case of cylindrical particles, \( A_{\text{min}} \) is equal to \( \pi d^{2}/4 \) and \( A_{\text{max}} \) is a function of \( E \) and is equal to \( Ld \) for a cylinder with \( E = \infty \).

As it can be seen in Fig. 10, for particle C1 no preferred orientation exists while particles C2 and C3 tend to be suspended with their maximum projected area in more than 60% of the frames. This was expected as previous studies mentioned that oscillation frequency of cylinders decreases with the increase of the aspect ratio.\(^3,13,15,17,19\) Using results of the PTV code, secondary motions of cylindrical and nonspherical particles, in general, can be quantified and used in future models to provide better estimations of particle drag coefficients. High values of \( A^{*} \) do not imply that cylinder axis is horizontal (normal to air flow direction) since \( A_{\text{max}} \) occurs in different axis angle for cylinders with different aspect ratios. To clarify this, variations of a projected area of the cylindrical particles versus their axis angle are presented in Fig. 11. This figure shows that the cylinder aspect ratio increases for maximum projected area associated with low angles. Comparing Figs. 10 and 11 it can be seen that in more than 60% of the cases particles C2 and C3 fall with \( A^{*} > 0.9 \), which corresponds to axis angles lower than 40°. In contrast, particle C1 falls with \( A^{*} > 0.6 \), which corresponds to axis angles between 5° and 70°. Based on the investigation of Chow and Adams’ cylinders with \( \sqrt{S/E} > 2.5 \) undergo tumbling motion when they freely fall. They mentioned that particles with oscillation angles greater than 60° will begin to transition to tumbling. This is similar to the result of our experiments since for all our cylindrical particles \((\sqrt{S/E} \geq 2)\), existence of \( A^{*} \) lower than 0.6 \((\alpha > 60°)\) indicates that the particles are going under tumbling motion.

VII. DISCUSSION AND CONCLUSION

Particle transport in either high particle to fluid density ratios or high particle Reynolds number is very common in a wide range of applications. Almost all the processes which involve transportation of solid particles in a gas continuum have very high values of density ratios. Existing literature on the drag coefficient of particles is mostly based on horizontal wind tunnel measurements on fixed bodies or freely falling particles in liquids. Data obtained with such methods cannot
be used to estimate drag coefficient of freely moving solid particles in gases.

In order to provide models on the drag coefficient of freely moving particles in high density ratios and high Reynolds numbers, a 4 m high vertical wind tunnel was built at the University of Geneva in collaboration with the fluid mechanics group (CMEFE) from the University of Applied Sciences Western Switzerland in Geneva (HES-SO/hepia). The wind tunnel is primarily designed to study drag coefficient of non-spherical particles especially highly irregular volcanic particles and can reach velocities between 5 and 27 ms⁻¹ (i.e., it can suspend volcanic particles between 10 and 40 mm which correspond to 5 × 10⁵ < Re < 8 × 10⁴, 500 kgm⁻³ < ρp < 2700 kgm⁻³).

Due to high level of particle secondary motions inside the test section, a PTV code is developed to calculate drag coefficient of particles from particle equation of motion (Eq. (4)). The input data to the PTV code include: (i) data provided by ImageJ on the particle centroid position in each frame, (ii) geometrical and physical properties of the particles resulted from 3D-scanning and laboratory measurements, (iii) database created from projected image of particle 3D-model in random orientations, (iv) equation of air velocity resulted from airflow calibration (Eq. (3)), and (v) data logged by temperature, pressure, and humidity sensors installed in the test section. The PTV code calculates absolute and relative velocity, acceleration of the particle, and thermodynamic properties of air during the experiment, particle shape descriptors based on random-orientation database, drag coefficient of the particle using existing models in the literature for benchmarking and drag coefficient of the particle measured in standard conditions using its real projected area as the reference area. By measuring real projected area of the suspended particles, the PTV code can provide very useful information on the secondary motions of particles and their preferred orientations which can affect particle drag coefficient.

The experiments are repeatable with the average relative standard deviation of 1.7%. Our measurements on the drag coefficient of spherical particles are in close agreement with previous studies. In case of cylindrical particles our results closely agree with results of Christiansen and Barker who measured drag coefficient of freely falling cylindrical particles in the air. On the other hand, significant difference between drag coefficient measured in our wind tunnel with those measured in horizontal wind tunnel and free fall in water is observed. This shows that the effects of density ratio and secondary motion of particles are very important on its drag coefficient. The PTV code performance on the calculation of real projected area of the particle is compared qualitatively with the descriptive reports available on the secondary motion of cylindrical particles and good agreements are found.

Results show that the tunnel design parameters and its calibration along with the PTV code can be used to produce reliable and accurate measurements of the drag coefficient of particles of various shapes. The wind tunnel is designed originally for the study of settling velocity and aggregation of volcanic particles, but it could also be used in various fields of multiphase flows that include fluid-particle or particle-particle interactions.

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